NAG C Library Function Document

nag_ztbtrs (f07vsc)

1 Purpose

nag_ztbtrs (f07vsc) solves a complex triangular band system of linear equations with multiple right-hand sides, $AX = B$, $A^T X = B$ or $A^H X = B$.

2 Specification

```
void nag_ztbtrs (Nag_OrderType order, Nag_UploType uplo, Nag_TransType trans,
     Nag_DiagType diag, Integer n, Integer kd, Integer nrhs, const Complex ab[],
     Integer pdab, Complex b[], Integer pdb, NagError *fail)
```

3 Description

nag_ztbtrs (f07vsc) solves a complex triangular band system of linear equations $AX = B$, $A^T X = B$ or $A^H X = B$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (1989) The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: indicates whether A is upper or lower triangular as follows:

if **uplo** = **Nag_Upper**, A is upper triangular;

if **uplo** = **Nag_Lower**, A is lower triangular.

Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.

3: **trans** – Nag_TransType *Input*

On entry: indicates the form of the equations as follows:

if **trans** = **Nag_NoTrans**, the equations are of the form $AX = B$;

if **trans** = **Nag_Trans**, the equations are of the form $A^T X = B$;

if **trans** = **Nag_ConjTrans**, the equations are of the form $A^H X = B$.

Constraint: **trans** = **Nag_NoTrans**, **Nag_Trans** or **Nag_ConjTrans**.

4: **diag** – Nag_DiagType *Input*

On entry: indicates whether A is a non-unit or unit triangular matrix as follows:

if **diag** = **Nag_NonUnitDiag**, A is a non-unit triangular matrix;

if **diag** = **Nag_UnitDiag**, A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = **Nag_NonUnitDiag** or **Nag_UnitDiag**.

5: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

6: **kd** – Integer *Input*

On entry: k, the number of super-diagonals of the matrix A if **uplo** = **Nag_Upper** or the number of sub-diagonals if **uplo** = **Nag_Lower**.

Constraint: **kd** ≥ 0.

7: **nrhs** – Integer *Input*

On entry: r, the number of right-hand sides.

Constraint: **nrhs** ≥ 0.

8: **ab**[dim] – const Complex *Input*

Note: the dimension, dim, of the array **ab** must be at least $\max(1, \textbf{pdab} \times \textbf{n})$.

On entry: the n by n triangular matrix A. This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements a_{ij} depends on the **order** and **uplo** parameters as follows:

if **order** = **Nag_ColMajor** and **uplo** = **Nag_Upper**,
a_{ij} is stored in **ab**$[k + i - j + (j-1) \times \textbf{pdab}]$, for $i = 1, \ldots, n$ and $j = i, \ldots, \min(n, i + k)$;

if **order** = **Nag_ColMajor** and **uplo** = **Nag_Lower**,
a_{ij} is stored in **ab**$[i - j + (j-1) \times \textbf{pdab}]$, for $i = 1, \ldots, n$ and $j = \max(1, i - k), \ldots, i$;

if **order** = **Nag_RowMajor** and **uplo** = **Nag_Upper**,
a_{ij} is stored in **ab**$[j - i + (i-1) \times \textbf{pdab}]$, for $i = 1, \ldots, n$ and $j = i, \ldots, \min(n, i + k)$;

if **order** = **Nag_RowMajor** and **uplo** = **Nag_Lower**,
a_{ij} is stored in **ab**$[k + j - i + (i-1) \times \textbf{pdab}]$, for $i = 1, \ldots, n$ and $j = \max(1, i - k), \ldots, i$.

9: **pdab** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix A in the array **ab**.

Constraint: **pdab** $\geq$ **kd** $+ 1$.

10: **b**[dim] – Complex *Input/Output*

Note: the dimension, dim, of the array **b** must be at least $\max(1, \textbf{pdb} \times \textbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \textbf{pdb} \times \textbf{n})$ when **order** = **Nag_RowMajor**.

If **order** = **Nag_ColMajor**, the (i, j)th element of the matrix B is stored in **b**$[(j-1) \times \textbf{pdb} + i - 1]$ and if **order** = **Nag_RowMajor**, the (i, j)th element of the matrix B is stored in **b**$[(i-1) \times \textbf{pdb} + j - 1]$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

11: **pdb** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = **Nag_ColMajor**, **pdb** $\geq \max(1, \mathbf{n})$;
if **order** = **Nag_RowMajor**, **pdb** $\geq \max(1, \mathbf{nrhs})$.

12: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

On entry, **kd** = ⟨*value*⟩.
Constraint: **kd** ≥ 0.

On entry, **nrhs** = ⟨*value*⟩.
Constraint: **nrhs** ≥ 0.

On entry, **pdab** = ⟨*value*⟩.
Constraint: **pdab** > 0.

On entry, **pdb** = ⟨*value*⟩.
Constraint: **pdb** > 0.

NE_INT_2

On entry, **pdab** = ⟨*value*⟩, **kd** = ⟨*value*⟩.
Constraint: **pdab** $\geq$ **kd** $+ 1$.

On entry, **pdb** = ⟨*value*⟩, **n** = ⟨*value*⟩.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = ⟨*value*⟩, **nrhs** = ⟨*value*⟩.
Constraint: **pdb** $\geq \max(1, \mathbf{nrhs})$.

NE_SINGULAR

The matrix A is singular.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham (1989).

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(k)\epsilon|A|,$$

$c(k)$ is a modest linear function of k, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(k)\,\mathrm{cond}(A, x)\epsilon, \quad \text{provided} \quad c(k)\,\mathrm{cond}(A, x)\epsilon < 1,$$

where $\mathrm{cond}(A, x) = \||A^{-1}|\,|A|\,|x|\|_\infty / \|x\|_\infty$.

Note that $\mathrm{cond}(A, x) \leq \mathrm{cond}(A) = \||A^{-1}|\,|A|\|_\infty \leq \kappa_\infty(A)$; $< \mathrm{cond}(A, x)$ can be much smaller than $< \mathrm{cond}(A)$ and it is also possible for $\mathrm{cond}(A^H)$, which is the same as $\mathrm{cond}(A^T)$, to be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_ztbrfs (f07vvc), and an estimate for $\kappa_\infty(A)$ can be obtained by calling nag_ztbcon (f07vuc) with **norm** = **Nag_InfNorm**.

8 Further Comments

The total number of real floating-point operations is approximately $8nkr$ if $k \ll n$.

The real analogue of this function is nag_dtbtrs (f07vec).

9 Example

To solve the system of equations $AX = B$, where

$$A = \begin{pmatrix} -1.94 + 4.43i & 0.00 + 0.00i & 0.00 + 0.00i & 0.00 + 0.00i \\ -3.39 + 3.44i & 4.12 - 4.27i & 0.00 + 0.00i & 0.00 + 0.00i \\ 1.62 + 3.68i & -1.84 + 5.53i & 0.43 - 2.66i & 0.00 + 0.00i \\ 0.00 + 0.00i & -2.77 - 1.93i & 1.74 - 0.04i & 0.44 + 0.10i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -8.86 - 3.88i & -24.09 - 5.27i \\ -15.57 - 23.41i & -57.97 + 8.14i \\ -7.63 + 22.78i & 19.09 - 29.51i \\ -14.74 - 2.40i & 19.17 + 21.33i \end{pmatrix}.$$

Here A is treated as a lower triangular band matrix with 2 sub-diagonals.

9.1 Program Text

```
/* nag_ztbtrs (f07vsc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
```

```
{
  /* Scalars */
  Integer  i, j, k, kd, n, nrhs, pdab, pdb;
  Integer  exit_status=0;
  Nag_UploType uplo_enum;
  NagError fail;
  Nag_OrderType order;

  /* Arrays */
  char    uplo[2];
  Complex  *ab=0, *b=0;

#ifdef NAG_COLUMN_MAJOR
#define AB_UPPER(I,J) ab[(J-1)*pdab + k + I - J - 1]
#define AB_LOWER(I,J) ab[(J-1)*pdab + I - J]
#define B(I,J) b[(J-1)*pdb + I - 1]
  order = Nag_ColMajor;
#else
#define AB_UPPER(I,J) ab[(I-1)*pdab + J - I]
#define AB_LOWER(I,J) ab[(I-1)*pdab + k + J - I - 1]
#define B(I,J) b[(I-1)*pdb + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);
  Vprintf("f07vsc Example Program Results\n\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vscanf("%ld%ld%ld%*[^\n] ", &n, &kd, &nrhs);
  pdab = kd + 1;
#ifdef NAG_COLUMN_MAJOR
  pdb = n;
#else
  pdb = nrhs;
#endif

  /* Allocate memory */
  if ( !(ab = NAG_ALLOC((kd+1) * n, Complex)) ||
       !(b = NAG_ALLOC(n * nrhs, Complex)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A from data file */
  Vscanf(" ' %1s '%*[^\n] ", uplo);
  if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
  else if (*(unsigned char *)uplo == 'U')
    uplo_enum = Nag_Upper;
  else
    {
      Vprintf("Unrecognised character for Nag_UploType type\n");
      exit_status = -1;
      goto END;
    }
  k = kd + 1;
  if (uplo_enum == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= MIN(i+kd,n); ++j)
            Vscanf(" ( %lf , %lf )", &AB_UPPER(i,j).re, &AB_UPPER(i,j).im);
        }
      Vscanf("%*[^\n] ");
    }
  else
    {
      for (i = 1; i <= n; ++i)
```

```
        {
          for (j = MAX(1,i-kd); j <= i; ++j)
            Vscanf(" ( %lf , %lf )", &AB_LOWER(i,j).re, &AB_LOWER(i,j).im);
        }
      Vscanf("%*[^\n] ");
    }
  /* Read B from data file */
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= nrhs; ++j)
        Vscanf(" ( %lf , %lf )", &B(i,j).re, &B(i,j).im);
    }
  Vscanf("%*[^\n] ");

  /* Compute solution */
  f07vsc(order, uplo_enum, Nag_NoTrans, Nag_NonUnitDiag, n,
         kd, nrhs, ab, pdab, b, pdb, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07vsc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Print solution */
  x04dbc(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, nrhs, b, pdb,
         Nag_BracketForm, "%7.4f", "Solution(s)", Nag_IntegerLabels,
         0, Nag_IntegerLabels, 0, 80, 0, 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from x04dbc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
 END:
  if (ab) NAG_FREE(ab);
  if (b) NAG_FREE(b);
  return exit_status;
}
```

9.2 Program Data

```
f07vsc Example Program Data
  4  2  2                                                  :Values of N, KD and NRHS
  'L'                                                       :Value of UPLO
 (-1.94, 4.43)
 (-3.39, 3.44) ( 4.12,-4.27)
 ( 1.62, 3.68) (-1.84, 5.53) ( 0.43,-2.66)
               (-2.77,-1.93) ( 1.74,-0.04) ( 0.44, 0.10)  :End of matrix A
 ( -8.86, -3.88) (-24.09, -5.27)
 (-15.57,-23.41) (-57.97,  8.14)
 ( -7.63, 22.78) ( 19.09,-29.51)
 (-14.74, -2.40) ( 19.17, 21.33)                           :End of matrix B
```

9.3 Program Results

```
f07vsc Example Program Results

 Solution(s)
                     1                   2
 1  ( 0.0000, 2.0000)  ( 1.0000, 5.0000)
 2  ( 1.0000,-3.0000)  (-7.0000,-2.0000)
 3  (-4.0000,-5.0000)  ( 3.0000, 4.0000)
 4  ( 2.0000,-1.0000)  (-6.0000,-9.0000)
```